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Collateral in Credit Rationing in Markets with Imperfect Information: Note

By HILDEGARD C. WETTE*

In their 1981 article, Joseph Stiglitz and Andrew Weiss analyze adverse selection and incentive effects in the loan market. The models considered are based on two crucial assumptions: borrowers are subject to limited liability; and lenders cannot distinguish borrowers (projects) of different risk. Stiglitz and Weiss show that a bank that raises its interest rate may suffer adverse selection because only risky borrowers will be willing to borrow at the higher rate. Thus lenders may choose not to raise the interest rate to eliminate excess demand, resulting in the possibility of a "credit rationing equilibrium." Stiglitz and Weiss also consider briefly the role of collateral in such credit rationing models. They conclude that lenders may choose not to use collateral requirements as a rationing device. An increase in collateral requirements, like an increase in the interest rate, potentially leads to a decrease in the lender's expected return on loans because of resulting adverse incentive and selection effects.

The purpose of this note is to further investigate the role of collateral in these models. Stiglitz and Weiss' discussion in Section III establishes that adverse selection effects can result from increases in collateral when borrowers are risk averse. I will show by returning to a model they discussed earlier in Section I, that the adverse selection effects can also occur when borrowers are risk neutral.

Stiglitz and Weiss outline a model to consider the use of collateral as a rationing device (Section III). In that model, all potential borrowers face the same array of risky projects; each potential borrower chooses (at most) one of those projects to undertake. The individuals are, by assumption, risk averse with decreasing absolute risk aversion, and possess different amounts of initial wealth. Thus, choice of project (if any) to undertake and the method of finance—self-finance from initial wealth or loan finance—will differ from one individual to the next. Stiglitz and Weiss show that, among those who undertake risky projects and who choose borrowing as the method of finance, wealthier individuals undertake riskier projects. An increase in collateral has two effects on the market for loans: those individuals who remain in the market will choose to undertake less-risky projects; and those individuals who drop out of the market are less-wealthy, low-risk borrowers. If the second effect is sufficiently strong, then increased collateral requirements will mean decreased expected returns for the lender. Thus, a credit rationing equilibrium may occur, since lenders may not choose to use collateral requirements (or the interest rate) to eliminate excess demand. The adverse selection effect just described does not occur in this model if the individuals are risk neutral. Consequently, the potential for a credit rationing equilibrium is limited to cases where borrowers are risk averse.

To see that increases in collateral requirements can also result in adverse selection if borrowers are risk neutral, consider the model Stiglitz and Weiss used to analyze the adverse selection effects of increases in the interest rate (Section I, pp. 395–99). It differs from the Section III model discussed above in three ways. First, borrowers are risk neutral; second, all projects ar loan financed. The analogy to the Section III model is that no individuals have sufficient wealth to self-finance projects.[1] Third, in the Section

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[1]As in all the Stiglitz-Weiss models, the choice of self-financing a portion of the project and loan financing the remainder is ruled out. It will become evident in what follows, however, that individuals are assumed to have enough wealth for the collateral required on loans.

III model, all borrowers face the same array of potential projects, and each borrower is "allowed" to undertake only one of them. In the Section I model, borrowers do not necessarily face the same set of project options, nor do they necessarily undertake only one. The lender knows that all available projects that yield nonnegative expective profit to borrowers at the given interest rate and collateral will be undertaken. (However, as in the Section III model, the lender cannot distinguish *which* borrower is undertaking *which* project.)

Each available project has a distribution of gross payoffs $F(R,\theta)$, where greater θ denotes greater risk. The lender is assumed to be able to evaluate the mean payoff of any project, but not its riskiness. The distribution of the riskiness of available projects with a given mean is denoted $G(\theta)$.[2] Stiglitz and Weiss show that, because of the limited liability assumption, the riskiest projects are the most profitable from the borrower's point of view. As the interest rate increases, the less-risky projects become unprofitable so borrowers do not undertake them. This result ultimately leads to the possibility of a credit rationing equilibrium.

In what follows, I use this model to show that increases in collateral requirements lead to adverse selection effects. Analysis parallels the Stiglitz-Weiss analysis concerning adverse selection effects of interest rate increases. My theorems differ from their theorems because collateral C, rather than the interest rate r, is allowed to vary.[3]

LEMMA: *Expected profit to the borrower is an increasing function of the riskiness of the project undertaken.*

PROOF:

As in Stiglitz and Weiss, the limited liability assumption implies that borrower profits on a particular project are a convex function of the gross payoff (R) of the project. Thus, increases in risk (θ) increase the expected profits of the borrower.

THEOREM 1-m: *For given collateral $\bar{C}$, there is a critical value $\tilde{\theta}$ such that an individual borrows from the bank (and undertakes the project) if and only if $\theta \geqslant \tilde{\theta}$.*

PROOF:

This follows directly from the lemma. Since the borrower's expected profits are an increasing function of θ, there is a level of θ $(\tilde{\theta})$ that yields zero expected profit. All projects with $\theta \geqslant \tilde{\theta}$ will thus be undertaken.

THEOREM 2-m: *As the collateral requirement increases, the critical value of θ below which individuals do not apply for loans increases.*

PROOF:

The value $\tilde{\theta}$ for which the borrower's expected profit is zero satisfies

$$\text{(5-m)} \quad \pi(\tilde{C},\tilde{\theta}) = -\tilde{C}\cdot F(1+r-\tilde{C},\tilde{\theta}) + \int_{1+r-C}^{\infty} [R-(1+r)]\, dF(R,\tilde{\theta}) = 0.$$

Total differentiation yields

(6-m)

$$\frac{d\tilde{\theta}}{d\tilde{C}} = -\left(\frac{\partial \pi(\tilde{C},\tilde{\theta})}{\partial C}\right) \Big/ \left(\frac{\partial \pi(\tilde{C},\tilde{\theta})}{\partial \tilde{\theta}}\right)$$

$$= F(1+r-\tilde{C},\tilde{\theta}) \Big/ \left(\frac{\partial \pi(\tilde{C},\tilde{\theta})}{\partial \tilde{\theta}}\right) > 0.$$

(We know from the lemma that $\partial\pi(\tilde{C},\tilde{\theta})/\partial\tilde{\theta} > 0$; also $F(1+r-\tilde{C},\tilde{\theta}) > 0$. Thus $d\tilde{\theta}/d\tilde{C} > 0$.)

Theorem 2-m is the crucial result. Increases in collateral will result in the less-risky (low θ) borrowers (projects) dropping out of the market for loans. The intuition is as follows. For a given project (given θ), an increase in collateral unambiguously increases the cost—and decreases the profit—to the borrower for some realizations of R. (Figure 1 illustrates this result.) Conse-

[2] More precisely, greater θ corresponds to greater risk in the sense of mean preserving spread as defined by Michael Rothschild and Stiglitz.

[3] The notation "m" is used to denote numbers of theorems and equations in Stiglitz-Weiss that have been altered due to my modifications. Clearly, in this analysis it must be assumed that all values of C under consideration are less than $(1+r)$; otherwise the lender would assume no risk in financing projects.

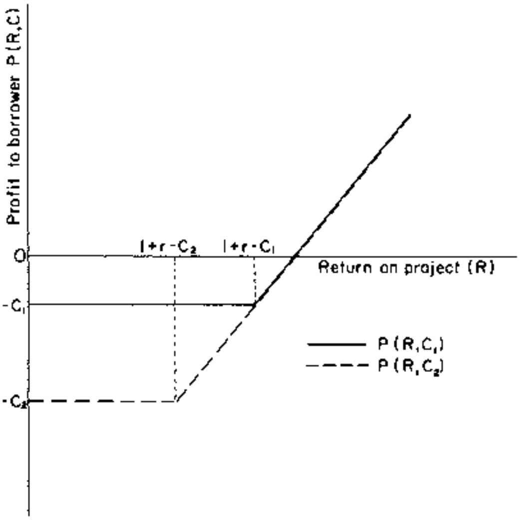

FIGURE 1

quently, expected profit for that project declines. This decrease in expected profit on each project means that some projects that were profitable at the initial level of C become unprofitable at the new, higher level of C. From the discussion of Theorem 1-m, we know those are the low-risk projects.

It remains only to show that this adverse selection effect from increases in collateral may cause a decrease in the lender's expected return on loans. The lender's expected return on a loan (project) of risk θ is $\rho(\theta,\tilde{C})$. Since the lender cannot determine which borrower undertakes the project θ, she must calculate an "average" expected return, $\bar{\rho}(\tilde{\theta}(\tilde{C}),\tilde{C})$.

(4b-m)

$$\rho(\theta,\tilde{C})=\int_0^{1+r-\tilde{C}}(R+\tilde{C})\,dF(R,\theta)+(1+r)\cdot(1-F(1+r-\tilde{C},\theta));$$

$$\text{(7-m)}\quad \bar{\rho}(\tilde{\theta}(\tilde{C}),\tilde{C})=\frac{\int_{\tilde{\theta}}^{\infty}\rho(\theta,\tilde{C})\,dG(\theta)}{1-G(\tilde{\theta})}.$$

We want to show that $\bar{\rho}$ is not necessarily an increasing function of C. This follows from differentiation of (7-m):

$$\text{(8-m)}\quad \frac{d\bar{\rho}}{d\tilde{C}}=\frac{\int_{\tilde{\theta}}^{\infty}F(1+r-\tilde{C},\theta)\,dG(\theta)}{1-G(\tilde{\theta})}+\frac{g(\tilde{\theta})}{1-G(\tilde{\theta})}(\bar{\rho}-\tilde{\rho})\frac{d\tilde{\theta}}{d\tilde{C}},$$

where $\tilde{\rho}=\rho(\tilde{\theta}(\tilde{C}),\tilde{C})$ is the expected return on the least-risky project undertaken. The first term in this expression is positive. The second term is negative because $d\tilde{\theta}/d\tilde{C}>0$ (by Theorem 2-m) and $\bar{\rho}<\tilde{\rho}$. The latter follows from Theorem 3 in Stiglitz-Weiss—the expected return on a loan to the lender is a decreasing function of the riskiness of the project financed by that loan; thus, the expected return to the lender on the least risky loan $\tilde{\rho}$ is greater than the expected return on the "average" loan $\bar{\rho}$ and $d\tilde{\theta}/d\tilde{C}>0$ (by Theorem 2-m).

Since $\bar{\rho}$, $\tilde{\rho}$, and $\tilde{\theta}$ do not depend on $g(\tilde{\theta})$, the negative term can be made arbitrarily large in absolute value by choosing $g(\tilde{\theta})$ large. Then, an increase in collateral reduces the lender's average expected return from loans.[4]

[4]The expressions for $d\tilde{\theta}/d\tilde{C}$ (equation (6-m)) and for $d\bar{\rho}/dC$ (equation (8-m)) look very similar to the analagous equations (6) and (8) in Stiglitz-Weiss for changes in r:

$$\text{(a)}\quad \frac{d\hat{\theta}}{d\hat{r}}=\frac{1-F((1+r)-C,\hat{\theta})}{\partial\pi(\hat{r},\hat{\theta})/\partial\hat{\theta}};$$

$$\text{(b)}\quad \frac{d\bar{\rho}}{d\hat{r}}=\int_{\hat{\theta}}^{\infty}\frac{[1-F(1+\hat{r}-C,\theta)]\,dG(\theta)}{1-G(\hat{\theta})}+\frac{g(\hat{\theta})}{1-G(\hat{\theta})}(\bar{\rho}-\hat{\rho})\frac{d\hat{\theta}}{d\hat{r}}.$$

The similarities occur because changes in C and in r both affect the cost of a given project to the borrower for certain realizations of R. In approximate terms, an increase in r affects the cost when the project "succeeds" (hence terms involving the upper tail of the project's distribution of returns, $1-F((1+r)-C,\theta)$); an increase in C affects the cost when the project "fails," hence terms involving the lower tail of the distribution, $F((1+r)-C,\theta)$.

The conclusion in the Stiglitz-Weiss paper is that if borrowers are risk neutral and subject to limited liability, then a lender may not be willing to increase the interest rate on loans to eliminate excess demand; if the borrowers are risk averse, he may also not be willing to raise collateral requirements to eliminate excess demand. The analysis here demonstrates that the lender also may not be willing to use collateral requirements as a rationing device even when borrowers are risk neutral, because increases in collateral can lead to adverse selection effects that decrease the lender's expected return on loans.

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